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## LASER-INDUCED TEMPERATURE RISE IN A COMPOSITE SANDWICH STRUCTURE

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**Abstract:** We investigate the transient temperature rise in a composite sandwich structure induced by a stationary, dithering, or rotating laser beam. We restrict our study to the composite sandwich structure with carbon fiber as skin materials and honeycomb as core materials. Our numerical simulations indicate that the maximum temperature rise behaves as a quadratic function of the reciprocal of the skin thickness or the effective beam size.

**Keywords:** dithering or rotating Gaussian laser beam; non-homogeneous heat equation; composite sandwich structure

**2010 AMS Subject Classification:** 35K05; 80A20.

### 1. Introduction

In recent years developing materials, such as materials of composite sandwich structure, to enhance the damage tolerance of the object has become more and more important. The goal of this paper is to investigate the maximum temperature rise of a composite sandwich structure induced by a stationary, rotating and dithering laser beam and to seek an optimal design of the composite sandwich structure.

The research on the temperature rise induced by a laser beam is not new. In the past decades many theoretical and numerical studies of temperature profiles induced by

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laser radiation in solids were conducted (Cline and Anthony, 1977; Lax, 1977 and 1978; Bertolotti and Sibilia, 1981; Burgener and Reedy, 1982; Calder and Sue, 1982; Moody and Hendel, 1982; Sanders, 1984; Araya and Gutierrez, 2006; Sistaninia et al., 2009). Temperature distributions in a two-layer structure by a scanning laser beam were investigated in (Burgener and Reedy, 1982). Recently, detailed studies of the temperature rise induced by a rotating or dithering laser beam were given on a semi-infinite domain (Zhou, 2011a), a solid with finite geometry (Zhou and Tan, 2011; Tan and Zhou, 2012), a two-layer structure (Zhou, 2011b; Zhou, 2012a) and a composite with a sandwich structure (Zhou, 2012b).

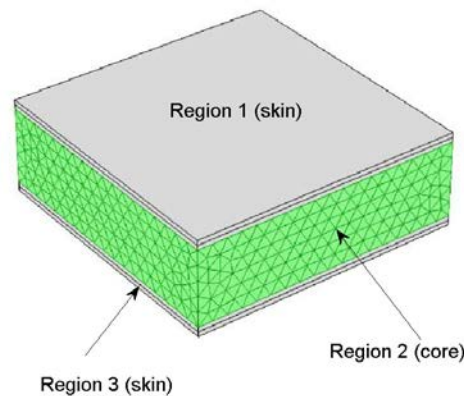
Composite materials are man-made materials and they are fabricated by attaching two stiff but thin skins to a thick, lightweight core. The thick core provides the sandwich-structured composite with high bending stiffness and overall low density. In (Zhou, 2012b), the performance of a sandwich-structured composite with different combinations of skin materials and core materials was analyzed. It was found that carbon fibers are better skin materials than E-glass fibers and composite sandwich structures with carbon fiber skin and honeycomb core can serve extremely well as heat shields. However, the scaling behavior of the maximum temperature rise versus different design parameters was left unexplored in (Zhou, 2012b). In this paper, we extend our earlier preliminary studies to investigate the effects of design parameters, including the thickness of the skin materials, the thickness of the core materials and the beam size, on the maximum temperature rise of the composite. Intuitively, the thicker the skin is, the smaller the maximum temperature rise will be and it would provide more heat shields. However, thicker skin will increase the density of the composite and eventually lose the attractive lightweight property. So there is a compromise between heat protection and lightweight. We wish to investigate the dependence of the maximum temperature rise of the structure on the thickness of the skin materials and provide insight on the design of the composite. Similar argument applies to the thickness of the core and we need to investigate its effect as well. In this paper we limit our study to a sandwich-structured composite made of honeycomb core sandwiched between two thin layers of carbon fibers since this selection of materials

has been shown to provide an effective heat shield from our previous studies (Zhou, 2012b).

We organize our paper as follows. In Sections 2-3 we present the modeling and numerical solutions of the temperature distributions in a three-dimensional composite sandwich structure induced by a stationary, dithering and rotating laser beam, respectively. Finally, we draw conclusions in Section 4.

## 2. MATHEMATICAL FORMULATIONS

We first briefly review a time-dependent mathematical model of a three-dimensional composite sandwich structure with one core material sandwiched between two thin layers of other materials. Laser beams hit the top surface of the sandwich structure. Figure 1 shows a schematic diagram of the composite sandwich structure. We divide the finite structure into three regions: Region 1 is a thin layer of thickness  $d_1$  in the  $z$  direction, which is on a substrate of dissimilar material of thickness  $d_2$  (Region 2), and Region 3 is another thin layer of thickness  $d_3$  at the bottom of the structure. The materials in both Region 1 and Region 3 are the same. Usually we set  $d_1 = d_3$  so the top and bottom skin layers share the same thickness.



**Figure 1: A schematic diagram of a three-dimensional sandwich-structured composite.**

The temperature distribution in the composite sandwich structure can be modeled by the standard non-homogeneous heat equation (John, 1981). More specifically, in Region 1 the governing equation is

$$\frac{\partial u_1}{\partial t} = \alpha_{T,1} \left( \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) + \frac{\alpha_{T,1}}{K_{T,1}} q(x, y, z, t), \quad (1)$$

where  $u_1(x, y, z, t)$  denotes the temperature rise at position  $(x, y, z)$  and time  $t$ ,  $\alpha_{T,1}$  is the thermal diffusivity of the material in Region 1,  $K_{T,1}$  is the thermal conductivity, and  $q(x, y, z, t)$  is the energy distribution of the incident laser beam. In the simulations presented here we model the distribution of a dithering or rotating Gaussian beam as

$$q(x, y, z, t) = f(x, y, t) \delta(z - z_0). \quad (2)$$

Here  $f(x, y, t)$  has the expression

$$\begin{aligned} f(x, y, t) &= \frac{I_0}{2\pi d^2} \exp \left[ -\frac{(x - x_c(t))^2 + (y - y_c(t))^2}{2d^2} \right], \\ x_c(t) &= x_0 + a \cos \frac{2\pi t}{T}, \quad x_0 = \frac{L_x}{2}, \\ y_c(t) &= y_0 + b \sin \frac{2\pi t}{T}, \quad y_0 = \frac{L_y}{2}, \end{aligned} \quad (3)$$

where  $(x_c(t), y_c(t), z_0)$  describes the position of the dithering or rotating Gaussian beam,  $(x_0, y_0, z_0)$  gives the initial position of the moving laser beam,  $I_0$  is the intensity of the laser beam,  $d$  is the effective radius of the laser beam, and  $\delta(z - z_0)$  is the Dirac delta function. Equation (3) represents a rotating laser beam when  $a = b$  and a dithering beam when either  $a$  or  $b$  is zero. When  $a = b = 0$ , equation (3) simply represents a non-moving stationary Gaussian beam.

In Region 2, where the material has the thermal diffusivity  $\alpha_{T,2}$  and thermal conductivity  $K_{T,2}$ , the governing equation for the temperature rise  $u_2(x, y, z, t)$  is given by

$$\frac{\partial u_2}{\partial t} = \alpha_{T,2} \left( \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial z^2} \right). \quad (4)$$

Similarly, in Region 3 the equation for the temperature rise  $u_3(x, y, z, t)$  is

$$\frac{\partial u_3}{\partial t} = \alpha_{T,3} \left( \frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} + \frac{\partial^2 u_3}{\partial z^2} \right). \quad (5)$$

At the interfaces between different regions/materials, we impose the physical condition that the energy is conserved for the heat flow across the interfaces:

$$\begin{aligned} K_{T,1} \frac{\partial u_1}{\partial z} &= K_{T,2} \frac{\partial u_2}{\partial z} \quad \text{at the interface between Regions 1 and 2} \\ K_{T,2} \frac{\partial u_2}{\partial z} &= K_{T,3} \frac{\partial u_3}{\partial z} \quad \text{at the interface between Regions 2 and 3} \end{aligned} \quad (4)$$

The initial condition for the temperature rises  $u_1(x, y, z, t), u_2(x, y, z, t), u_3(x, y, z, t)$  is zero to reflect the fact that initially, the composite sandwich structure shares the same temperature as the ambient. The boundary conditions at the air/material interface require that the sandwich structure is insulated at all edges. The boundary conditions take the assumption that no energy leaks into the ambient at the air/material interface. This is fairly a good approximation for most materials under consideration because heat flow by conduction through the material is usually much larger than heat loss by radiation or convection at the air/material interface.

For the top (Region 1) and bottom (Region 3) layers of the composite sandwich structure, the materials are chosen to be *carbon fibers*. Carbon fibers are well-known for their high flexibility, high tensile strength, low weight, high temperature tolerance and low thermal expansion. They are widely used in aerospace, civil engineering, military, and automobile. For Region 2 we select lightweight, flame resistant *Nomex honeycomb* as the core material. The corresponding thermal properties of carbon fibers and Nomex honeycomb are given in Table 1 (Wang et al. 2007; Silva et al. 2005).

It is worthwhile to note that the skin materials have much larger thermal conductivity than the core materials.

**Table 1: Thermal properties for carbon fibers and Nomex honeycomb**

Materials	Heat Capacity (or Specific Heat) $C_p$ (unit: J/(kg*K))	Density $\rho$ (unit: Kg / m <sup>3</sup> )	Thermal Conductivity $K_T$ (unit: W/(m*K))	Thermal Diffusivity $\alpha_T = \frac{K_T}{\rho \cdot C_p}$ (unit: m <sup>2</sup> / s)
<b>Carbon fibers</b>	676	1760	84	$7.0602 \times 10^{-5}$
<b>Honeycomb</b>	1172 ~ 1340	1100	0.175	$1.1872 \times 10^{-7} \sim 1.3574 \times 10^{-7}$

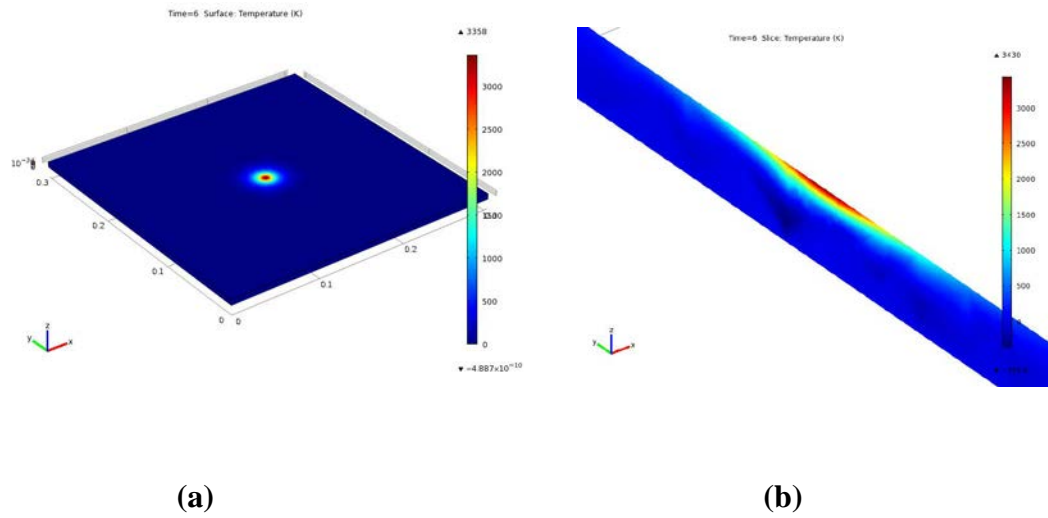
### 3. NUMERICAL RESULTS

In this section we investigate the effects of different design parameters on the maximum temperature rise of a sandwich-structured composite. We apply the commercial software COMSOL to solve the partial differential equations (1)-(4) with the prescribed initial and boundary conditions. COMSOL is a multi-physics commercial software based on finite element methods which can be used to solve our model equations efficiently.

We first construct a sandwich structure where the core has thickness  $d_2$  (mm) and cross section dimension  $304.8mm \times 304.8mm$  and the top and bottom skins are with thickness  $d_1 = d_3$  (mm). In our study we will fix  $d_2$  and vary  $d_1$  and then we will fix  $d_1$  and vary  $d_2$ .

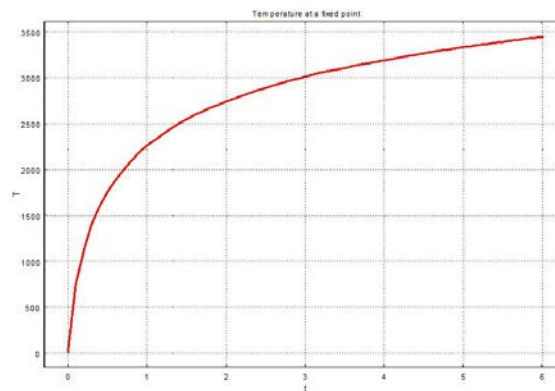
We start with a stationary Gaussian beam applied on the top surface of the sandwich structure where  $I_0 = 500W, d = 0.0025m, a = b = 0, L_x = L_y = 0.3048m$  in (3). In Figure 2(a) we take a snapshot of the temperature rise distribution of the composite structure after the laser beam is shone for 6 seconds on the top surface whereas in

Figure 2(b) we show the temperature rise in a vertical slice across the center of the top surface.



**Figure 2: The temperature rise of a sandwich structure induced by a stationary Gaussian beam. (a) a 3-D view; (b) a vertical slice through the center of the top surface.**

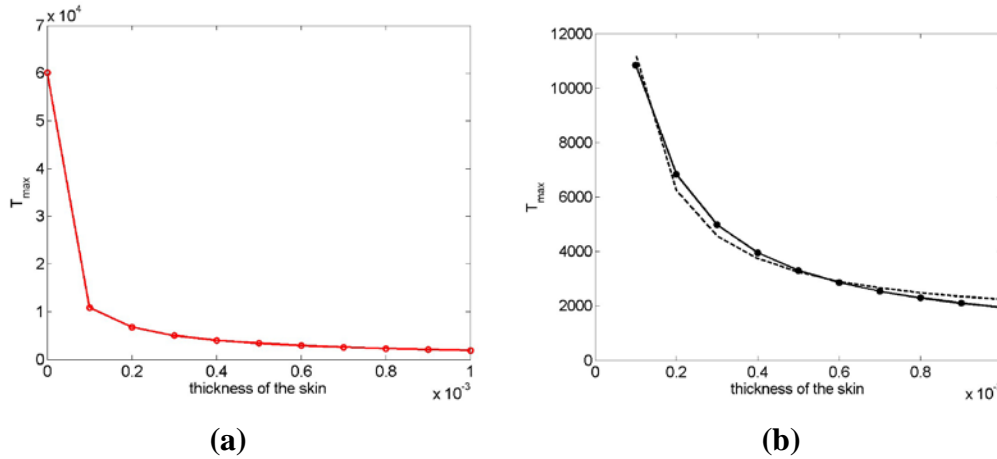
From Figure 2 we see that the maximum temperature (around 3358K) occurs at the center of the top surface which is exactly the spot where the laser beam hits the structure. In Figure 3 we plot the transient temperature rise at the center of the top surface. As time goes by, the spot gets hotter and hotter. Here the thickness of the core is fixed as  $d_2 = 6mm$  and the skin thickness is chosen as  $d_1 = d_3 = 0.5mm$ .



**Figure 3: The transient temperature rise of a sandwich structure at the center of the top surface.**



Now we vary the thickness of the skin from 0mm (no skin) to 1mm and compute the maximum temperature rise in the composite structure. The maximum temperature rise versus the thickness of the skin is plotted in Figure 4(a).



**Figure 4: (a) The maximum temperature rise of a sandwich structure as a function of the skin thickness. (b) The data in Figure 4(a) fitted by two different least squares functions.**

Note that even with a thin layer of skin on the top and bottom of the core, the maximum temperature rise can be reduced significantly in comparison with the case where there is no skin at all. So even with a thin layer of skin one gains big protection. In Figure 4(b) we fit the data in Figure 4(a) by two different sets of least squares fitting functions. The dashed curve corresponds to the fitting function

$$T_{\max}^{\text{fitted}} = \frac{1.001}{d_1} + 1233.3 \quad \text{whereas the solid curve is described by}$$

$$T_{\max}^{\text{fitted}} = \frac{-0.000047}{d_1^2} + \frac{1.5}{d_1} + 486.0 . \quad \text{The maximum temperature rise is fitted better by}$$

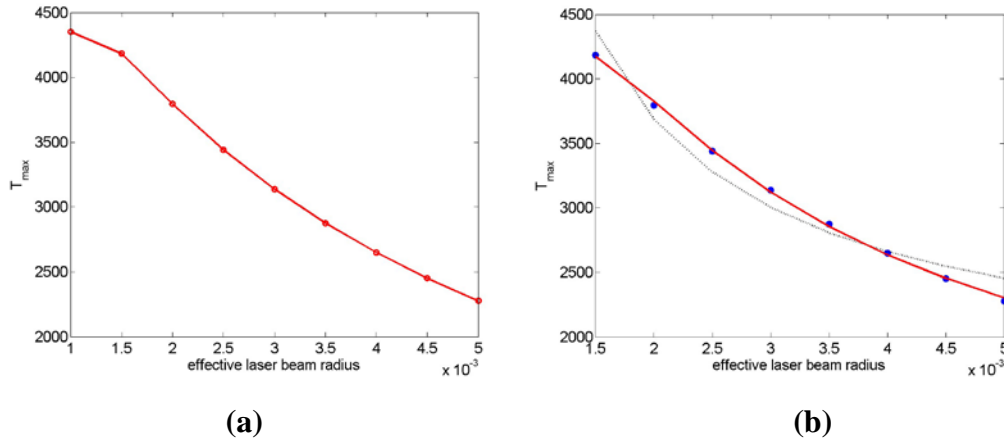
the second fitting function which is a quadratic function of the reciprocal of the skin thickness.

A practical interpretation of Figure 4 can be as follows. If we want to build an object with sandwich structured composite materials which can survive laser attacks

(with power 500W) for six seconds with a maximum temperature rise less than, say, 4000K, then the skin thickness should be greater than 0.4mm.

Another possible way to enhance damage tolerance is to increase the core thickness while fixing the skin thickness. Intuitively, increasing core thickness will enlarge the bulk volume of the materials and therefore is expected to reduce the maximum temperature rise at a fixed time. We carried out some preliminary simulations and our results indicate that changing the core thickness is not significant in reducing the maximum temperature rise.

Now we would like to see how the beam size affects the maximum temperature rise of the composite. We fix the composite structure so that the top and bottom skin layers have thickness  $d_1 = d_3 = 0.5$  mm and the middle core has thickness  $d_2 = 6$  mm while varying the effective beam radius  $d$  from 1mm to 5mm. Figure 5(a) depicts the maximum temperature rise as a function of the effective beam radius whereas Figure 5(b) shows two sets of least squares fittings with the data in Figure 5(a).



**Figure 5: (a) The maximum temperature rise of a sandwich structure versus the effective beam radius of a stationary Gaussian beam. (b) The data in Figure 6(a) fitted by two different least squares functions.**

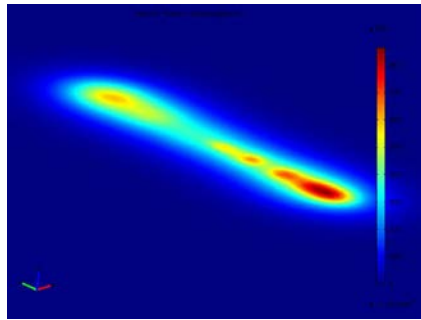
More specifically, in Figure 5(b) the dotted curve is fitted by the function

$$T_{\max}^{\text{fitted}} = \frac{4.1}{d} + 1637.3 \quad \text{while the solid curve is fitted by the function}$$

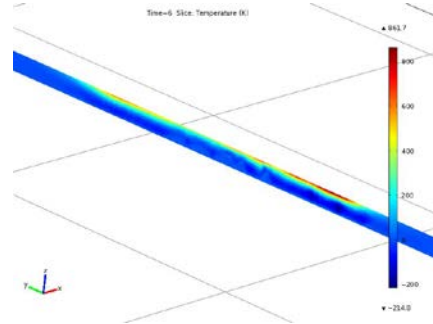
$$T_{\max}^{\text{fitted}} = \frac{-0.0064}{d^2} + \frac{9.5590}{d} + 646.1341. \quad \text{It appears that the solid curve fits the data}$$

better than the dotted curve. Figure 5 shows that the maximum temperature rise decreases when the beam size increases. This is expected since the beam becomes less focused when its size increases.

Next we study the temperature rise induced by a dithering laser beam. In formula (3) we choose  $I_0 = 500W, d = 0.0025m, a = 0, b = 0.05m, L_x = L_y = 0.3048m, T = 1s$ . In Figure 6 we give a snapshot of the temperature rise at final time=6s of the top surface (left panel) and the vertical slice passing through the center of the top surface (right panel). Due to the dithering nature of the beam, the maximum temperature rise occurs at different spots at different times. The maximum temperature rise induced by a dithering beam is dropped significantly to 862K compared with the stationary beam case where the maximum temperature rise is around 3358K.



(a)

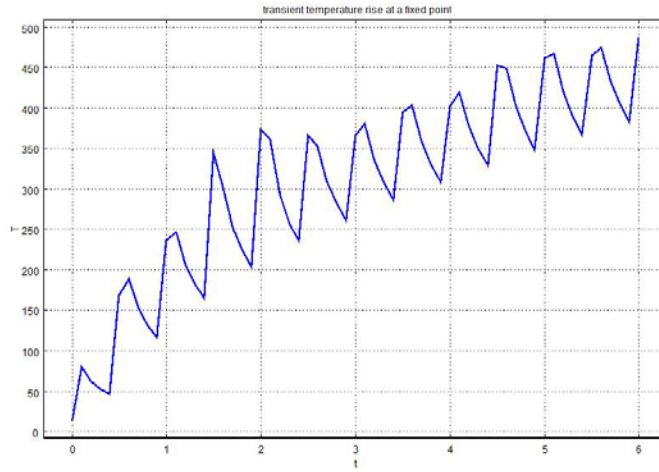


(b)

**Figure 6: The maximum temperature rise of a sandwich structure induced by a dithering laser beam. Left panel: top surface; right panel: a vertical slice.**

We also follow the transient temperature rise at a fixed point (the center of the top surface). The result is shown in Figure 7. Due to the fact that the beam moves back

and forth along a line and passes through the center periodically, the temperature rise at a fixed point oscillates and the envelope of the amplitude increases with time.



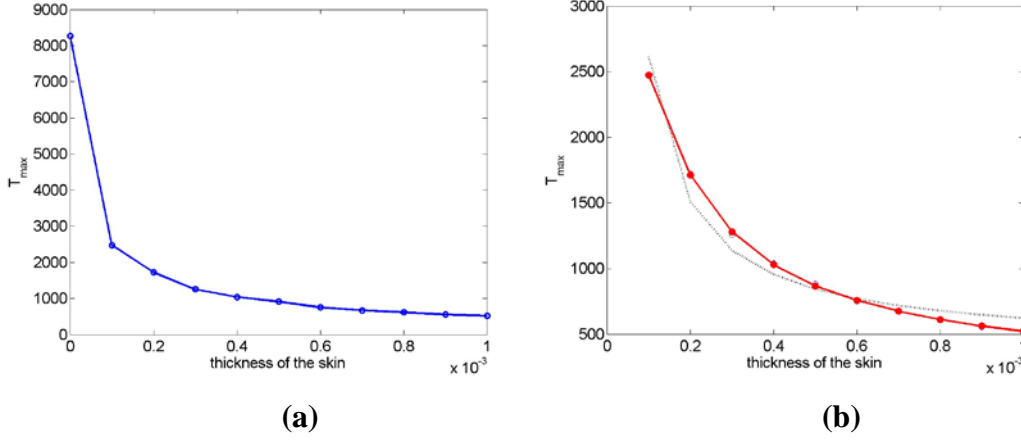
**Figure 7: The transient dithering-laser-beam-induced temperature rise of a sandwich structure at the center of the top surface.**

For a dithering laser beam, we will investigate the effect of the skin thickness as well as the effect of the beam size on the maximum temperature rise. From the previous study of the stationary beam, the maximum temperature rise is not very sensitive to the core thickness and therefore will not be studied any more. In Figure 8(a) we show the maximum temperature rise dependence on the thickness of the skin layer while in Figure 8(b) we fit the data with two different least squares functions. The dotted curve

corresponds to the fitted function  $T_{\max}^{\text{fitted}} = \frac{0.2210}{d_1} + 401.7029$  whereas the solid

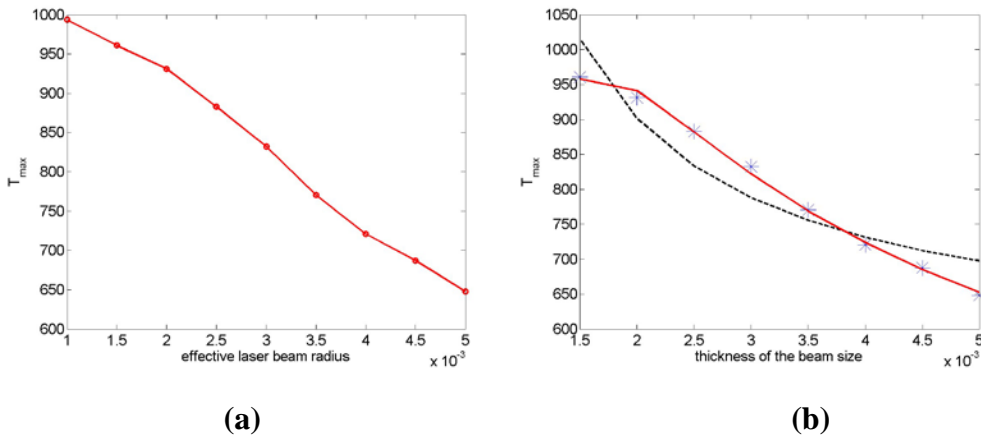
curve is described by  $T_{\max}^{\text{fitted}} = \frac{-0.000016}{d_1^2} + \frac{0.395}{d_1} + 143.20$ . As expected, a

combination of sandwich structure and dithering beam leads to a big reduction in the maximum temperature rise.



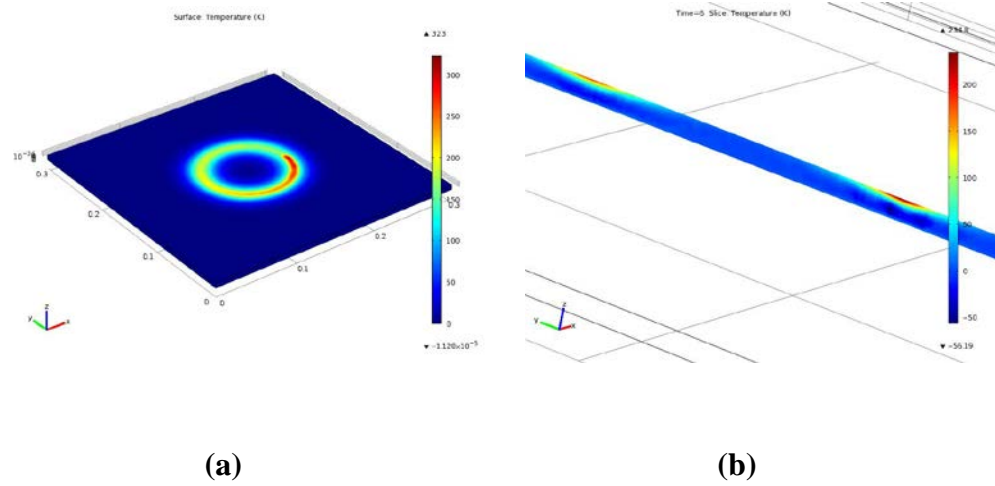
**Figure 8: (a) The dithering-beam-induced maximum temperature rise of a sandwich structure as a function of the skin thickness. (b) The data fitted by two different least squares functions.**

In Figure 9(a) we plot the maximum temperature rise as a function of the effective laser beam radius. As the beam size increases, the maximum temperature rise decreases. We fit the data in Figure 9(a) by two different functions in Figure 9(b). Here the dashed curve is described by a linear function of the reciprocal of the beam radius  $T_{\max}^{\text{fitted}} = \frac{0.6795}{d} + 561.4320$  and the solid curve is given by a quadratic function of the reciprocal of the beam radius  $T_{\max}^{\text{fitted}} = \frac{-0.0018}{d^2} + \frac{2.2564}{d} + 275.2048$ . Clearly, the maximum temperature rise behaves more like a quadratic function of the reciprocal of the beam size.



**Figure 9: (a) The maximum temperature rise of a sandwich structure versus the effective beam radius of a dithering Gaussian beam. (b) The data fitted by two different least squares functions.**

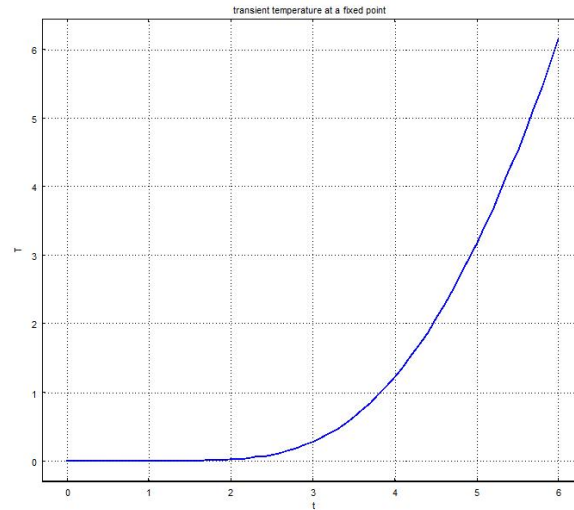
Finally we move to the temperature rise induced by a rotating laser beam. First in Figure 10(a) we take a snapshot of the temperature rise induced by a rotating laser beam with  $I_0 = 500W, d = 0.0025m, a = b = 0.05m, L_x = L_y = 0.3048m$  in expression (3).



**Figure 10: The maximum temperature rise of a sandwich structure induced by a rotating laser beam. Left panel: top surface; right panel: a vertical slice.**

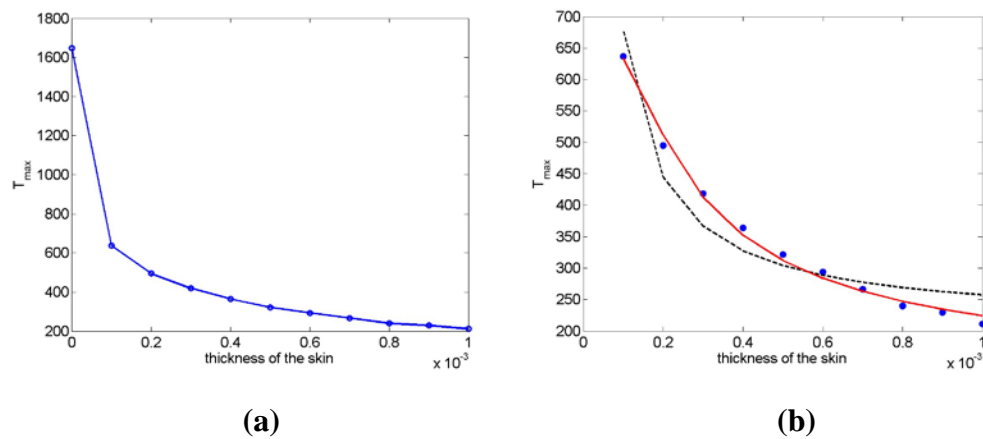
Figure 10(a) indicates that the maximum temperature rise drops further to about 323K. Figure 10(b) shows the temperature rise on a vertical slice.

In Figure 11 we plot the transient temperature rise at the center point of the top surface. Here the temperature rise increases monotonically as a function of time even though the magnitude of the temperature rise is very small. The increasing nature of the temperature is due to the insulating boundary condition.



**Figure 11: The transient rotating-laser-beam-induced temperature rise of a sandwich structure at the center point of the top surface.**

Now we want to see how the maximum temperature rise behaves as we vary either the skin thickness or the effective beam size. Our numerical results are shown in Figures 12 and 13, respectively, together with least squares fitting functions.

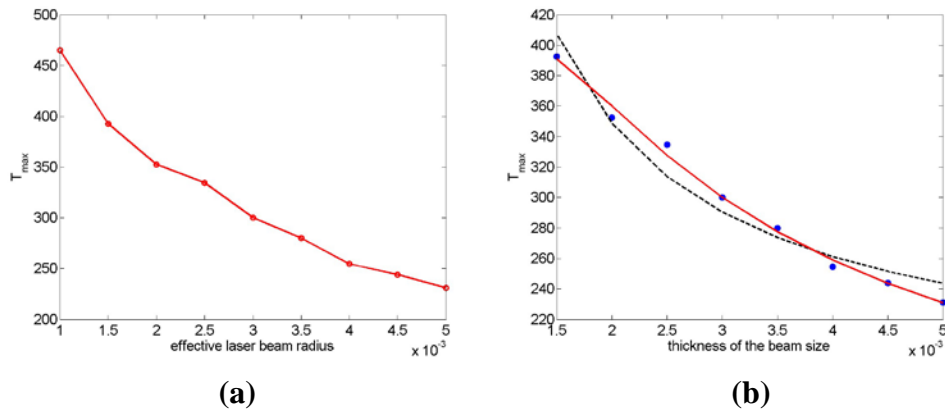


**Figure 12: (a) The rotating-beam-induced maximum temperature rise of a sandwich structure as a function of the skin thickness  $d_1$ . (b) The data in Figure 12(a) fitted by two different least squares functions.**

In Figure 12, the dashed curve is given by  $T_{\max}^{\text{fitted}} = \frac{0.0469}{d_1} + 210.4757$

whereas the solid curve is  $T_{\max}^{\text{fitted}} = \frac{-0.000005}{d_1^2} + \frac{0.104}{d_1} + 125.98$ .

Figure 12 and Figure 13 again demonstrate that the maximum temperature rise can be described by a quadratic function of the reciprocal of the skin thickness or the beam size where the coefficients in the quadratic form are different for each case.



**Figure 13: (a) The maximum temperature rise of a sandwich structure versus the effective beam radius of a rotating Gaussian beam  $d$ . (2) The data in Figure 13(a) fitted by two different least squares functions. The dashed curve is described by**

$$T_{\max}^{\text{fitted}} = \frac{0.35}{d} + 173.65 \quad \text{and the solid curve is} \quad T_{\max}^{\text{fitted}} = \frac{-0.000527}{d^2} + \frac{0.7998}{d} + 92.01.$$

#### 4. CONCLUDING REMARKS

We have shown numerically how the maximum temperature rise induced by a laser beam for a composite sandwich structure scales as a function of the thickness of the skin layer or the effective beam size. Our study includes three kinds of laser beams: a stationary Gaussian beam, a dithering Gaussian beam or a rotating Gaussian beam. All the cases share a universal rule: the maximum temperature rise behaves as a quadratic function of the reciprocal of the skin thickness or the effective beam size. Validation of our predictions with the experimental data will be pursued in the future.



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